Neutrino kinetics in a magnetized dense plasma

V.N. Oraevsky, V.B. Semikoz
The Institute of Terrestrial Magnetism, Ionosphere and
Radio Wave Propagation of the Russian Academy of Sciences,
IZMIRAN, Troitsk, Moscow region, 142190, Russia

The relativistic kinetic equations (RKE) for lepton plasma in the presence of a strong external magnetic field are derived in Vlasov approximation. The new RKE for the electron spin distribution function includes the weak interaction with neutrinos originated by the axial vector current ($\sim c_A$) and provided by the parity nonconservation. The isotropic medium limit with zero magnetic field is checked against recent developments of the dispersion equation in an isotropic plasma with the neutrino component. The clear form of the neutrino RKE accounting for the neutrino electromagnetic formfactors in an isotropic plasma with the quasi-static and high-frequency self-consistent electromagnetic field limits that lead to the appearance of the corresponding *induced electric charges* of neutrino is presented. In a polarized electron gas Bloch equation describing the evolution of the magnetization density perturbation is derived from the electron spin RKE being modified in the presence of neutrino fluxes. Such modified hydrodynamical equation allows to obtain the new dispersion equation in a magnetized plasma from which *the neutrino driven instability of spin waves* can be found. It is shown that this instability is more efficient than the analogous one for Langmuir waves enhanced in an isotropic plasma.

I. INTRODUCTION

When he conjectured the existence of the neutrino, Pauli imposed very stringent bounds on its electrical neutrality. Nevertheless, the direct (=weak) interaction of neutrino with electrons and positrons shows a principle possibility of the interaction of neutrino with the electromagnetic field. However, in vacuum this interaction is negligible. The situation is extremely changed in media with free carriers of the electric charge such as a dense plasma of metals, stars, the lepton plasma of the early Universe, etc.

It was shown in [1–3] that the electrodynamics of neutrinos is changed in such media comparing with the electrodynamics of neutrinos in vacuum. The main distinction is the appearance of *the induced electric charge of neutrino* which is proportional to the density of free carriers of electricity [1]. Namely this leads to the new qualitative effect: the appearance of the long-distance forces inevitably leads to the collective interactions of neutrino with an ensemble of charged particles through electromagnetic field.

The approximation of the neutrino propagation in an isotropic plasma considered in [1–6] and then in [7] has some natural bounds for applications in astrophysics. Therefore one needs to have the self-consistent system of kinetic equations of the most general kind applicable for other astrophysical plasmas including magnetized stars.

Really this occurs a new problem unsolved before in the neutrino kinetics while in the one-particle approach using the Schrödinger equation it has been already demonstrated the importance of the axial vector potential for the neutrino propagation in a magnetized plasma ($V_A \sim <\bar{\psi}_e\gamma_i\gamma_5\psi_e> \sim B_i$) that provides the interaction of neutrino with the magnetic field $\mathbf{B}$ without any neutrino magnetic moment [8] and could explain, e.g. the neutron star kick [9].

In the present work in equations for the self-consistent fields and particles we have taken into account the spin interactions of electrons that are important for neutrino propagation in a dense degenerate electron gas polarized by the external magnetic field.

In section 2 we have presented the full set of the coupled Relativistic Kinetic Equations (RKE) for electrons and neutrinos in the lepton plasma with an external magnetic field including the self-consistent electromagnetic field contribution. These RKE are derived using the Bogolyubov method analogously to the way described in details for an isotropic plasma in [6].

In subsection 2.1 we discuss the isotropic limit for the neutrino RKE explaining the physical sense of the damping force which is linear over the Fermi constant G_F and in some limits can be connected with an induced electric charge

---

[1]There appears also *induced magnetic moment of neutrino* [4], cross-sections of the neutrino scattering off nuclei are modified in plasmas especially in low energy region [5] and so on.

of neutrino in an isotropic plasma. Then in the same subsection we reproduce the dispersion equation accounting also for the $\nu\nu$- and ee- weak scattering through $Z^{(0)}$-boson [6] and comparing this equation with the recent formulae obtained in [7,10].

In subsection 2.2. we discuss the equilibrium state in an electron gas polarized by the external magnetic field $\mathbf{B}_0 \neq 0$.

In section 3 we have generalized Bloch equation for the evolution of the magnetization density perturbation in the presence of neutrino fluxes. In section 4 neglecting spatial dispersion we obtain the increment of spin waves excited by the neutrino beam and compare it with the case of the neutrino driven instability of plasma waves in an isotropic plasma.

In section 5 we discuss results and give some estimates of relevant parameters in a polarized medium.

II. RELATIVISTIC KINETIC EQUATIONS FOR LEPTON PLASMA IN VLASOV APPROXIMATION

The kinetic equations in the Standard Model (SM) are derived from the quantum Liouville equation for the non-equilibrium statistical operator $\hat{\rho}(t)$ using the Bogolyubov method with the total interaction Hamiltonian given by the Feynman diagrams for the neutrino scattering off electrons and the usual ee-scattering in QED [6]. For a magnetized plasma we do not consider $\nu\nu$-scattering and have neglected weak ee-scattering comparing with the electromagnetic interaction of charged particles through the photon exchange.

The one-particle density matrix $\hat{f}^{(a)}_{r'r}(\mathbf{p},\mathbf{x},t) = \sum_{\mathbf{k}} e^{i\mathbf{kx}} Tr(\hat{\rho}(t)\hat{b}^{+(a)}_{\mathbf{p}-\mathbf{k}/2,r}\hat{b}^{(a)}_{\mathbf{p}+\mathbf{k}/2,r'})$ is determined as the statistical average of the product of creation ($\hat{b}^{+(a)}_{\mathbf{p}r}$) and annihilation ($\hat{b}^{(a)}_{\mathbf{p}'r'}$) operators for the lepton $a = e, \nu$. In the case of electrons it takes of the form

$$\hat{f}^{(e)}_{\lambda'\lambda}(\mathbf{p}_e,\mathbf{x},t) = f^{(e)}(\mathbf{p}_e,\mathbf{x},t)\frac{\delta_{\lambda'\lambda}}{2} + S^{(e)}_i(\mathbf{p}_e,\mathbf{x},t)\frac{(\sigma_i)_{\lambda'\lambda}}{2} ,$$

where $\lambda = \pm 1$ is the eigenvalue of the spin projection on an external magnetic field. E.g. for the magnetic field $\mathbf{B} = (0,0,B_0)$ only the spin component Σ_z commutes with the Hamiltonian for electrons, $[\Sigma_z, \hat{\mathcal{H}}^{(e)}]_= 0$, hence $(\sigma_z)_{\lambda'\lambda} = \lambda\delta_{\lambda'\lambda}$. Not assuming a neutrino magnetic moment the one-particle density matrix for massless Dirac neutrinos corresponds to the active (left-handed, $r = -1$) neutrino distribution function only,

$$\hat{f}^{(\nu)}_{r'r}(\mathbf{q},\mathbf{x},t) = \frac{(1-r)\delta_{r'r}}{2} f^{(\nu)}(\mathbf{q},\mathbf{x},t) ,$$

where in an uniform medium the helicity $r = \pm 1$ conserves, $(\sigma_i q_i)_{r'r} = qr\delta_{r'r}$.

The RKE for the Wigner number density distribution functions $f^{(a)}(\mathbf{p},\mathbf{x},t) = Tr\left[\hat{f}^{(a)}(\mathbf{p},\mathbf{x},t)\right]$ that are Lorentz-invariant take of the covariant forms

$$q_\mu \frac{\partial f^{(\nu)}(\mathbf{q},\mathbf{x},t)}{\partial x_\mu} + \frac{G_F c_V}{\sqrt{2}} \int \frac{d^3 p_e}{(2\pi)^3} \frac{(p^\mu_e q_\mu)}{\varepsilon_{p_e}} \frac{\partial f^{(e)}(\mathbf{p}_e,\mathbf{x},t)}{\partial x_j} \frac{\partial f^{(\nu)}(\mathbf{q},\mathbf{x},t)}{\partial q_j} +$$
$$+ \frac{G_F c_A m_e q^\mu}{2\sqrt{2}} \int \frac{d^3 p_e}{(2\pi)^3 \varepsilon_{p_e}} \frac{\partial a^{(e)}_\mu(\mathbf{p}_e,\mathbf{x},t)}{\partial x_j} \frac{\partial f^{(\nu)}(\mathbf{q},\mathbf{x},t)}{\partial q_j} = 0 , \qquad (1)$$

for neutrinos and

$$p_\mu \frac{\partial f^{(e)}(\mathbf{p},\mathbf{x},t)}{\partial x_\mu} + eF_{j\mu}(\mathbf{x},t)p^\mu \frac{\partial f^{(e)}(\mathbf{p},\mathbf{x},t)}{\partial p_j} +$$
$$+ \frac{G_F c_V}{\sqrt{2}} \int \frac{d^3 q}{(2\pi)^3} \frac{(p^\mu q_\mu)}{\varepsilon_q} \frac{\partial f^{(\nu)}(\mathbf{q},\mathbf{x},t)}{\partial x_j} \frac{\partial f^{(e)}(\mathbf{p},\mathbf{x},t)}{\partial p_j} +$$
$$+ \frac{G_F c_A m_e}{\sqrt{2}} \int \frac{d^3 q}{(2\pi)^3} \frac{(q_\mu a^\mu_i(\mathbf{p}))}{\varepsilon_q} \frac{\partial S^{(e)}_i(\mathbf{p},\mathbf{x},t)}{\partial p_j} \frac{\partial f^{(\nu)}(\mathbf{q},\mathbf{x},t)}{\partial x_j} = 0 , \qquad (2)$$

for electrons where $c_V = 2\xi \pm 0.5$, $c_A = \mp 0.5$ are the vector and axial weak couplings with upper (lower) sign for electron (muon or tau) neutrinos correspondingly .

Note that in the presence of the external magnetic field $\mathbf{B}_0$ the neutrino parts of these Vlasov equations differ from the analogous ones used in [7] due the inclusion of axial vector interactions ($\sim c_A$) in the total νe-scattering amplitude that allows us to consider also *collective interactions of ν_μ and ν_τ-neutrinos* for which the vector coupling c_V is small, $c_V = 2\xi - 0.5 \approx 0$.

Incorporating the weak interaction of electrons with neutrinos we have derived the RKE for the electron spin distribution function $\mathbf{S}^{(e)}(\mathbf{p},\mathbf{x},t)$,

$$\frac{\partial S_i^{(e)}(\mathbf{p},\mathbf{x},t)}{\partial t} + \mathbf{v}\frac{\partial S_i^{(e)}(\mathbf{p},\mathbf{x},t)}{\partial \mathbf{x}} + eF_{j\mu}(\mathbf{x},t)\frac{p^\mu}{\varepsilon_p}\frac{\partial S_i^{(e)}(\mathbf{p},\mathbf{x},t)}{\partial p_j} +$$
$$+2\mu_B\left[\frac{E_i(\mathbf{x},t)(\mathbf{v}\mathbf{S}^{(e)}(\mathbf{p},\mathbf{x},t)) - v_i(\mathbf{E}(\mathbf{x},t)\mathbf{S}^{(e)}(\mathbf{p},\mathbf{x},t))}{1+\gamma} + \frac{[\mathbf{S}^{(e)}(\mathbf{p},\mathbf{x},t)\times\mathbf{B}(\mathbf{x},t)]}{\gamma}\right] -$$
$$-\frac{G_F c_A m_e}{\sqrt{2}\varepsilon_p}\int\frac{d^3q}{(2\pi)^3}\frac{(q_\mu a_i^\mu(\mathbf{p}))}{\varepsilon_q}\frac{\partial f^{(e)}(\mathbf{p},\mathbf{x},t)}{\partial p_j}\frac{\partial f^{(\nu)}(\mathbf{q},\mathbf{x},t)}{\partial x_j} = 0\ , \tag{3}$$

that is the relativistic generalization in SM of the kinetic equation for spin waves in nonferromagnetic metals [11] while we have considered here the Fermi gas of free electrons in contrast with the quasi-particle approach for metals and neglected also exchange interactions (neglecting exchange Feynman diagrams for the ee-scattering meaning here that the long-range forces are dominant). Moreover, we assume the quasi-classical electron spectrum in a realistic external magnetic field B_0 obeying the inequality $eB_0 \ll T^2$, or the Landau spectrum $\varepsilon_p(\lambda,n) = (m_e^2 + p_z^2 + \mid e \mid B_0(2n+1-\lambda))^{1/2}$ contains the small paramagnetic (spin) correction to the continuous spectrum $\varepsilon_p = \sqrt{p^2+m_e^2}$,

$$\varepsilon_p(\lambda) = \varepsilon_p - \frac{\lambda \mid e \mid B_0}{2\varepsilon_p}\ , \tag{4}$$

where $\lambda = \pm 1$ and we have changed $\mid e \mid B_0(2n+1) = p_\perp^2$ for large Landau numbers $n \gg 1$.

The system of RKE's is completed by the Maxwell equations for the electromagnetic field $F_{\mu\nu}(\mathbf{x},t)$ accounting for the spin wave contribution,

$$-\frac{1}{c}\frac{\partial \mathbf{B}(\mathbf{x},t)}{\partial t} = [\nabla \times \mathbf{E}(\mathbf{x},t)]\ ,$$

and

$$[\nabla\times[\nabla\times\mathbf{E}(\mathbf{x},t)]] + \frac{1}{c^2}\frac{\partial^2\mathbf{E}(\mathbf{x},t)}{\partial t^2} = -\frac{4\pi}{c^2}\frac{\partial}{\partial t}\Bigl(e\int\frac{d^3p}{(2\pi)^3}\mathbf{v}f^{(e)}(\mathbf{p},\mathbf{x},t) +$$
$$+\mu_B\left[\int\frac{d^3p}{(2\pi)^3}\frac{[\nabla\times\mathbf{S}^{(e)}(\mathbf{p},\mathbf{x},t)]}{\gamma} - \int\frac{d^3p}{(2\pi)^3}(\mathbf{v}\nabla)\frac{[\mathbf{v}\times\mathbf{S}^{(e)}(\mathbf{p},\mathbf{x},t)]}{\gamma+1}\right]\Bigr)\ . \tag{5}$$

In eqs. (1)–(5) $e = -\mid e \mid$ is the electric charge of the electron; G_F is the Fermi constant; $\mu_B = e\hbar/2m_ec$ is the Bohr magneton; $\gamma = \varepsilon_e/m_e$ is the γ-factor using here and in the following units $\hbar = c = 1$; the latin indices run $i,j = 1,2,3$ and the greek ones are $\mu = 0,1,2,3$ for scalars written in the Feynman metrics, $A_\mu B^\mu = A_0B_0 - A_iB_i$.

In the non-relativistic (NR) limit $\mid \mathbf{v} \mid \ll 1$ ($\gamma \to 1$) the last term drops and eq. (5) coincides with the Maxwell equation in a magnetized medium:

$$[\nabla\times(\mathbf{B}-4\pi\mathbf{M})] = 4\pi\mathbf{j} + \frac{\partial\mathbf{E}}{\partial t}\ , \tag{6}$$

where $\mathbf{M}(\mathbf{x},t) = \mu_B\int d^3p\mathbf{S}(\mathbf{p},\mathbf{x},t)/(2\pi)^3$ is the magnetization density of NR plasma.

Multiplying eq. (3) by the energy ε_p and the spin distribution function $S_i(\mathbf{p},\mathbf{x},t)$ we obtain the covariant RKE for the Lorentz-invariant product $a_\mu(\mathbf{p},\mathbf{x},t)a^\mu(\mathbf{p},\mathbf{x},t) = -(\mathbf{S}^{(e)}(\mathbf{p},\mathbf{x},t))^2$ where we have introduced in eq. (1) the 4-vector $a_\mu^{(e)}(\mathbf{p},\mathbf{x},t) \equiv a_\mu^i(\mathbf{p})S^i(\mathbf{p},\mathbf{x},t)$ that is the statistical generalization [6] of the Pauli-Lubański 4-vector $a^\mu = (a_i^\mu(\mathbf{p})\zeta_i)$ and has the components

$$a_\mu^{(e)}(\mathbf{p},\mathbf{x},t) = \left[\frac{\mathbf{p}\mathbf{S}^{(e)}(\mathbf{p},\mathbf{x},t)}{m_e};\mathbf{S}^{(e)}(\mathbf{p},\mathbf{x},t) + \frac{\mathbf{p}(\mathbf{p}\mathbf{S}^{(e)}(\mathbf{p},\mathbf{x},t))}{m_e(\varepsilon_e+m_e)}\right]\ .$$

Note that for the uniform electron beam ($S_i^{(e)}(\mathbf{p},t) = S_i(t)(2\pi)^3 n_{e0}\delta^{(3)}(\mathbf{p}-\mathbf{p}_0)$) omitting the neutrino term and integrating the spin eq. (3) over d^3p we obtain the one-particle spin evolution equation

$$\frac{d\mathbf{S}(t)}{dt} = \frac{2\mu_B}{\gamma}[\mathbf{S}\times\mathbf{B}] + \frac{2\mu_B}{\gamma+1}\Bigl[\mathbf{S}\times\Bigl[\mathbf{E}\times\mathbf{v}\Bigr]\Bigr]\ ,$$

that occurs exactly the Bargman-Mishel-Telegdi equation for the electron spin motion with the *normal* magnetic moment μ_B [12]. In general, it is possible to generalize eq. (3) adding the electromagnetic scattering of electrons through *anomalous* magnetic moment (Schwinger correction $\mu' = (\alpha/2\pi)\mu_B$) that could lead to additional terms similar to electromagnetic terms in the neutrino spin RKE [13].

A. Isotropic medium limit

Here we reproduce some results obtained by authors for the case $\mathbf{B}_0 = 0$ and developed in literature for the last decade identifying these old formulae with the analogous ones derived in numerous recent references and giving some extension of the dispersion equation [7,10].

a) Induced electric charge and damping force for neutrinos

Neglecting weak interactions for electrons and substituting the solution of the Boltzman equation (2) obtained in the linear approximation $f^{(e)}(\mathbf{p},\mathbf{x},t) = f_0^{(e)}(\varepsilon_p) + \delta f^{(e)}(\mathbf{p},\mathbf{x},t)$ into the neutrino RKE (1) one can derive the tractable neutrino kinetic equation in the collisionless Vlasov approximation [6,2]

$$\frac{\partial f^{(\nu)}(\mathbf{q},\mathbf{x},t)}{\partial t} + \mathbf{n}\frac{\partial f^{(\nu)}(\mathbf{q},\mathbf{x},t)}{\partial \mathbf{x}} + e\int \frac{d^4Q e^{-iQx}}{(2\pi)^4}\Big[F_l(\omega,k)\mathbf{E}_{\parallel}(\omega,\mathbf{k}) + F_{tr}(\omega,k)) \times$$
$$\times\Big(\mathbf{E}_{tr}(\omega,\mathbf{k}) + [\mathbf{n}\times\mathbf{B}(\omega,\mathbf{k})]\Big)\Big]\frac{\partial f^{(\nu)}(\mathbf{q},\mathbf{x},t)}{\partial \mathbf{q}} = 0\ , \tag{7}$$

where $F_l(\omega,k)$ and $F_{tr}(\omega,k)$ are the neutrino electromagnetic formfactors,

$$\begin{aligned} F_l(\omega,k) &= G_F\sqrt{2}c_V(\varepsilon_l(\omega,k)-1)Q^2/\alpha\ , \\ F_{tr}(\omega,k) &= G_F\sqrt{2}c_V(\varepsilon_{tr}(\omega,k)-1)\omega^2/\alpha\ , \end{aligned} \tag{8}$$

given by the longitudinal and transversal permittivities $\varepsilon_{l,tr}(\omega,k)$ of an isotropic plasma and derived in [6] using the RKE approach and using the Finite Temperature Field Theory (FTFT) method in [14] for the neutrino electromagnetic vertex.

Here $Q_\mu = (\omega,\mathbf{k})$ is the four momentum transfer; the unit vector $\mathbf{n} = \mathbf{q}/q$ is the velocity of massless neutrino ; $\mathbf{E} = \mathbf{E}_{\parallel} + \mathbf{E}_{\perp}$, $\mathbf{B}$ are the electromagnetic fields in the dispersive medium, and $\mathbf{E}_{\parallel} = \mathbf{k}(\mathbf{kE})/k^2$.

In the FTFT approach the same result for the third term in the RKE eq. (7) can be obtained from the statistical QED loop correction to the neutrino four-vector potential in plasma

$$\delta V_\mu^{(V)}(\mathbf{x},t) = \frac{G_F\sqrt{2}c_V}{4\pi\alpha}j_\mu^{ind}(\mathbf{x},t) = \frac{G_F\sqrt{2}c_V}{4\pi\alpha}\int\frac{d^4Q}{(2\pi)^4}e^{-iQx}\Pi_{\mu\nu}(\omega,\mathbf{k})\delta A^\nu(\omega,\mathbf{k})$$

after the substitution of this correction into the force term of the neutrino RKE,

$$eF_{i\mu}^{(V)}(\mathbf{x},t)\frac{q_\mu}{q}\frac{\partial f^{(\nu)}(\mathbf{q},\mathbf{x},t)}{\partial q_i}\ ,$$

given by the antisymmetric tensor $F_{i\mu}^{(V)} = \partial\delta V_\mu^{(V)}(\mathbf{x},t)/\partial x_i - \partial\delta V_i^{(V)}(\mathbf{x},t)/\partial x_\mu$.

It is obvious that the third term on the left-hand side of eq. (7) is proportional to the force of electromagnetic origin. Whereas for a point charge e (when the form factors are equal to unity: $F_l = F_{tr} = 1$) this term is determined by the Lorentz force, i.e., is equal to the standard expression $e(\mathbf{E}(\mathbf{x},t) + [\mathbf{n}\times\mathbf{B}(\mathbf{x},t)])\partial f(\mathbf{q},\mathbf{x},t)/\partial\mathbf{q}$, for neutrinos with allowance for the Fermi constant to the electric charge, $G_F \sim e^2/M_W^2$, the third term in eq. (7) is proportional to the radiative damping force ($\sim e^3$).

The polarization origin of such a force becomes obvious after simple manipulations in eq. (7) using the explicit expressions (8) for the form factors F_l and F_{tr} in an isotropic dispersive medium, for which the Fourier integrals can be completely calculated, and the considered term [6]

$$\frac{\sqrt{2}G_Fc_V}{e}\left(\frac{\partial^2}{\partial x_j\partial_n} + n_n\frac{\partial^2}{\partial t\partial x_j}\right)P_n(\mathbf{x},t)\frac{\partial f^\nu(\mathbf{q},\mathbf{x},t)}{\partial q_i} \tag{9}$$

is proportional to the second derivative of the polarization vector of the dispersive medium:

$$4\pi P_n(\mathbf{x},t) = D_n(\mathbf{x},t) - E_n(\mathbf{x},t) \ ,$$

which is equal to the difference between the vectors of the electric displacement, $D_n(\mathbf{x},t) = \int d^4x'\varepsilon_{nj}(\mathbf{x}-\mathbf{x}',t-t')E_j(\mathbf{x}',t)$, and the electric field intensity $E_n(\mathbf{x},t)$.

In vacuum (since $\mathbf{D}-\mathbf{E}=0$ neglecting vacuum polarization), the effect corresponding to plasmon emission by the moving neutrino ($\nu\to\nu+\gamma^*$) disappears, i.e., there is no damping force in eq. (7).

Note that this damping force coincides with the result in [7] if we use the standard Maxwell equations in macroscopic electrodynamics [15] $\partial_n P/\partial x_n = -\rho = -e\delta n_e(\mathbf{x},t) = -e\int d^3p\delta f_e(\mathbf{p},\mathbf{x},t)/(2\pi)^3$ and $\partial P_n/\partial t = j_n(\mathbf{x},t) = e\int d^3pv_n\delta f_e(\mathbf{p},\mathbf{x},t)/(2\pi)^3$ changing the expression (9) to

$$\frac{\sqrt{2}G_Fc_V}{e}\nabla_j(-\rho+\mathbf{nj})\frac{\partial f^{(\nu)}(\mathbf{q},\mathbf{x},t)}{\partial\mathbf{q}} = -\sqrt{2}G_Fc_V\nabla_j\delta n_e(\mathbf{x},t)[1-\mathbf{nV}(\mathbf{x},t)]\frac{\partial f^{(\nu)}(\mathbf{q},\mathbf{x},t)}{\partial\mathbf{q}},$$

where $\mathbf{V}$ is the electron fluid velocity. This expression follows also from Eq. (17') in the ref. [6].

Finally, in special cases of the excitation in a dispersive medium of electrostatic waves ($\omega\ll k<v>$) or the propagation of a high-frequency transverse wave ($\omega\gg k<v>$) the force term (9) can be represented in the form of the effective Lorentz force [2]

$$e_\nu^{ind}\mathbf{E}_\parallel\frac{\partial f^{(\nu)}(\mathbf{q},\mathbf{x},t)}{\partial\mathbf{q}}$$

or

$$\tilde{e}_\nu^{ind}\left[\mathbf{E}+[\mathbf{n}\times\mathbf{B}]\right]\frac{\partial f^{(\nu)}(\mathbf{q},\mathbf{x},t)}{\partial\mathbf{q}} \ ,$$

which is proportional to the neutrino induced electric charges, to the quasi-static one [1], $e_\nu^{ind} = -|e|G_Fc_V/(2\pi\alpha\sqrt{2}r_D^2)$, or to $\tilde{e}_\nu^{ind} = -|e|G_Fc_V\omega_p^2/(2\pi\alpha\sqrt{2})$ [16] given by the Debye radius r_D or by the Langmuir (plasma) frequency $\omega_p=<v>/r_D$ correspondingly.

b) Dispersion equation for longitudinal waves in isotropic lepton plasma

Adding the $\nu\nu$- and ee-scattering through $Z^{(0)}$-boson in the interaction Hamiltonian and omitting terms with spin waves for the case $\mathbf{B}_0=0$, after the Fourier transformation $f^{(a)}(\mathbf{p},\mathbf{x},t) = \int d^4Qe^{-iQx}f^{(a)}(\mathbf{p},\mathbf{k},\omega)/(2\pi)^4$ with the following division of the neutrino and electron kinetic equations by $q(\omega-\mathbf{kn})$ and $\varepsilon_p(\omega-\mathbf{kv})$ correspondingly one can derive the dispersion equation for longitudinal waves in the lepton plasma as the determinant $D=0$ of the algebraic system for the Fourier transforms of the densities $n^\nu(\mathbf{k},\omega) = \int d^3qf^{(\nu)}\mathbf{q},\mathbf{k},\omega)/(2\pi)^3$ and $n^{(e)}(\mathbf{k},\omega) = \int d^3pf^{(e)}(\mathbf{p},\mathbf{k},\omega)/(2\pi)^3$ (Eq. (34) of the Ref. [6]):

$$\begin{aligned}&\varepsilon_l^A\left(\varepsilon_l^\nu-\frac{M_Z^2}{Q^2}\right)+\frac{(1-4\xi)^2}{4\sin^2 2\theta_W}\times(\varepsilon_l^A-1)-\\&-\frac{Q^2}{M_Z^2\sin^2 2\theta_W}\times(\varepsilon_l^A-1)(\varepsilon_l^\nu-1)\times\left[\frac{(1-4\xi)^2}{4}-c_V^2\right]=0\ .\end{aligned} \tag{10}$$

Here

$$\varepsilon_l^A \equiv 1+\chi_e(\omega,\mathbf{k}) = 1+\frac{4\pi\alpha}{k^2}\int\frac{d^3p}{(2\pi)^3}\frac{\mathbf{k}\partial f_0^{(e)}(\varepsilon_p)/\partial\mathbf{p}}{(\omega-\mathbf{kv})} \ ,$$

$$\varepsilon_l^\nu = 1+\frac{4\pi\alpha}{k^2\sin^2 2\theta_W}\int\frac{d^3q}{(2\pi)^3}\frac{\mathbf{k}\partial f_0^{(\nu)}(q)/\partial\mathbf{q}}{(\omega-\mathbf{kn})}$$

are the longitudinal permittivities for the electron and neutrino components correspondingly; $\xi=\sin^2\theta_W\sim 0.23$ is the SM (Weinberg) parameter.

For comparison with results [7,10] in the case of electron neutrinos ($c_{Ve}=0.5+2\xi$) this dispersion equation can be rewritten as

$$1+\chi_e(\omega,\mathbf{k})-\frac{k^2\Delta_\nu}{\omega_p^2}\left(1-\frac{\omega^2}{k^2}\right)^2(4\xi)\chi_e(\omega,\mathbf{k})\int\frac{d^3q}{(2\pi)^3}\frac{\mathbf{k}\partial\hat{f}_0^{(\nu)}(q)/\partial\mathbf{q}}{(\omega-\mathbf{kn})}-$$
$$-\frac{(1-4\xi)^2}{4\pi\alpha}G_F\sqrt{2}(\omega^2-k^2)\chi_e(\omega,\mathbf{k})-\Big(1+\chi_e(\omega,\mathbf{k})\Big)\sqrt{2}G_F\left(\frac{\omega^2}{k^2}-1\right)\times$$
$$\times n_0^{(\nu)}\int\frac{d^3q}{(2\pi)^3}\frac{\mathbf{k}\partial\hat{f}_0^{(\nu)}(q)/\partial\mathbf{q}}{(\omega-\mathbf{kn})}=0\ , \tag{11}$$

where $G_F=2\sqrt{2}\pi\alpha/M_Z^2\sin^2 2\theta_W$ is the Fermi constant, $\Delta_\nu=2G_F^2n_{e0}n_{\nu0}/m_e$ is the weak parameter introduced in [7], $\hat{f}_0^{(a)}=f_0^{(a)}(\varepsilon_a)/n_0^{(a)}$ are the equilibrium distribution functions normalized on the corresponding density and having the dimension $[\hat{f}]=M^{-3}$; $\omega_p=\sqrt{4\pi\alpha n_{e0}/m_e}$ is the non-relativistic expression for the plasma frequency.

The new (linear over G_F) terms are stipulated by the neutral current contributions coming in the fourth term from the weak ee-scattering and due to the $\nu\nu$-scattering in the last term. Unfortunately, these new terms lead to the negligible displacement of the resonant frequency while the increment is still determined by the first line in eq. (11) originated by the $\nu_e e$-scattering. The physical sense of such displacement is obvious: $\nu\nu$-scattering does not transfer energy to the electron subsystem as well as the weak ee-scattering to the neutrino one.

For $\nu_{\mu,\tau}$-neutrinos any instability effect is absent when one neglects the axial coupling c_A as we did in this subsection. Really, omitting the W-boson contribution (changing $(1+4\xi)^2/4=c_{Ve}^2$-term to $(1-4\xi)^2/4=c_{V\mu}^2$ within the brackets of the dispersion equation (10)) we loose the neutrino beam influence the plasma environment. The presence of the misleading coefficient $c_V^{'2}$ ahead Δ_ν in the work [10] is due to the neglect there the weak ee-scattering through the $Z^{(0)}$-boson (note that the vector part of this weak ee- amplitude is proportional to the same $(1-4\xi)$). This ee-scattering term compensates the neutral current contribution in eq. (10) originated by the νe-scattering.

For the case $\xi=1/4$ omitting also the last $\nu\nu$-scattering term in (11) we have reproduced exactly the dispersion equation obtained in [7]. The prediction of the streaming instability driven by the neutrino beam in a dense plasma (i.e., outside the neutrinosphere of a supernova) [7] was criticized in [10,17,18], however, in this section we do not touch these issues following from eq. (11).

Let us turn to the case of magnetized plasma.

B. Equilibrium state in a polarized electron gas

In the linear approximation the distribution functions are of the form

$$\begin{aligned}f^{(\nu)}(\mathbf{q},\mathbf{x},t)&=f_0^{(\nu)}(\mathbf{q})+\delta f^{(\nu)}(\mathbf{q},\mathbf{x},t)\ ,\\ f^{(e)}(\mathbf{p},\mathbf{x},t)&=f_0^{(e)}(\varepsilon_p)+\delta f^{(e)}(\mathbf{p},\mathbf{x},t)\ ,\\ \mathbf{S}^{(e)}(\mathbf{p},\mathbf{x},t)&=\mathbf{S}_0^{(e)}(\varepsilon_p)+\delta\mathbf{S}^{(e)}(\mathbf{p},\mathbf{x},t)\ ,\end{aligned} \tag{12}$$

where the neutrino background is given by $f_0^{(\nu)}(\mathbf{q})$ not being in equilibrium with a plasma environment (e.g. for the powerful neutrino flux outside of the SN neutrinosphere), and we have considered the uniform polarized equilibrium electron gas for which the one-particle Wigner density matrix takes of the form

$$\hat{f}_{\lambda'\lambda}^{(0e)}(\varepsilon_p)=\frac{\delta_{\lambda'\lambda}}{2}f^{(0)}(\varepsilon_p)+\frac{(\sigma_j)_{\lambda'\lambda}}{2}S_j^{(0)}(\varepsilon_p)\ . \tag{13}$$

Here $f^{(0)}(\varepsilon_p)=[\exp(\varepsilon_p-\zeta)/T)+1]^{-1}$ is the equilibrium Fermi function, and

$$S_j^{(0)}(\varepsilon_p)=-\frac{\mid\mu_B\mid B_j^{(0)}}{\gamma}\frac{df^{(0)}(\varepsilon_p)}{d\varepsilon_p} \tag{14}$$

is the equilibrium spin distribution.

One can easily check that in the quasi-classical limit given by eq. (26) the exact expression for the magnetization of the electron gas

$$M_j^{(0)}=\mu_B<\bar{\psi}_e\gamma_j\gamma_5\psi>_0=\mu_B\sum_{n=0}^{\infty}\frac{eB}{(2\pi)^2}\int_{-\infty}^{+\infty}dp_zTr(\sigma_jf_0^{(e)}(\varepsilon_p(\lambda,n)) \tag{15}$$

acquires the quasi-classical form

$$M_j^{(0)} = \mid \mu_B \mid \int \frac{d^3p}{(2\pi)^3} S_j^{(0)}(\varepsilon_p) = -2\mu_B^2 B_{0j} \int D(\varepsilon_p) \frac{df^{(0)}(\varepsilon_p)}{d\varepsilon_p} d\varepsilon_p \ , \qquad (16)$$

where the spin distribution $S_j^{(0)}(\varepsilon_p)$ is given by eq. (14) and $D(\varepsilon_p) = pm_e/(2\pi)^2$. In particular, in the NR limit this background magnetization corresponds to the spin paramagnetism of the free electron gas in metal [19].

On the other hand, in the same NR limit integrating eq. (3) over d^3p one can easily reproduce Bloch equation [20] for the magnetization density perturbation $\mathbf{m}(\mathbf{x}, t) = \mu_B \int d^3p \delta \mathbf{S}(\mathbf{p}, \mathbf{x}, t)/(2\pi)^3$ generalized in [11] for an inhomogeneous medium (see next section).

Note that for the degenerate electron gas from the quasi-classical Eq. (16) one obtains the same value $M_z^{(0)} = \mid \mu_B \mid\mid e \mid B_0 p_{F_e}/2\pi^2$ as it follows from the exact quantum eq. (15) for an arbitrary strong magnetic field for which electrons which populate the main Landau level (n=0) contribute only.

This magnetization determines also the axial vector potential of a probing neutrino in a magnetized plasma [8],

$$V_A = -G_F\sqrt{2}c_A \frac{q_j}{q} \frac{M_j^{(0)}}{\mid \mu_B \mid} \ ,$$

that changes the spectrum of the ultrarelativistic neutrino, $\varepsilon_q = q + V + V_A$, comparing with the standard one in an isotropic medium, $\varepsilon_q = q + V$, and modifies the neutrino oscillations in SN [8,9]. In our kinetic approach we do not consider oscillations assuming the spectrum $\varepsilon_q = q$ for massless neutrino of the given kind ν_a, $a = e, \mu, \tau$.

III. BLOCH EQUATION IN THE PRESENCE OF NEUTRINO BEAM

Substituting the linear decomposition (12) into eq. (3), then integrating the latter over d^3p and multiplying by μ_B we have generalized Bloch equation for the magnetization density perturbations $\mathbf{m}(\mathbf{x}, t)$ in a polarized NR electron gas that is the base of the theory of the electron paramagnetic resonance in the absence of weak interactions [20] while in SM with neutrinos such equation takes of the form

$$\frac{\partial m_j(\mathbf{x}, t)}{\partial t} + 2\mu_B \left[\Big(\mathbf{m}(\mathbf{x}, t) - \chi_0 \mathbf{b}(\mathbf{x}, t)\Big) \times \mathbf{B}_0\right]_j + \frac{\partial \Sigma_{kj}(\mathbf{x}, t)}{\partial x_k} + \\ + \frac{G_F c_A \mu_B n_{0e}}{\sqrt{2} m_e} \int \frac{d^3q}{(2\pi)^3} \frac{\partial \delta f^{(\nu)}(\mathbf{q}, \mathbf{x}, t)}{\partial x_j} = 0 \ . \qquad (17)$$

Here $\chi_0 = -2\mu_B^2 \int d^3p (df^{(0)}/d\varepsilon_p)/(2\pi)^3$ is the static susceptibility of the polarized electron gas [2]; $\mathbf{b}(\mathbf{x}, t)$ is the magnetic field perturbation in the total field $\mathbf{B}(\mathbf{x}, t) = \mathbf{B}_0 + \mathbf{b}(\mathbf{x}, t)$; $n_{e0} = \int d^3p f^{(0)}(\varepsilon_p)/(2\pi)^3$ is the density of the electron gas. The pseudotensor $\Sigma_{kj}(\mathbf{x}, t) = \mu_B \int d^3p v_k \delta S_j(\mathbf{p}, \mathbf{x}, t)/(2\pi)^3$ describes spatial inhomogeneity of the magnetization and the last *new vector* term ($\sim c_A$) corresponds to the parity violation for the evolution of the macroscopic *axial* vector m_j in SM.

In order to break the chain of hydrodynamical equations for magnetization moments, m_j, Σ_{ij}, etc, one can consider long-wave spin waves with the wave lengths λ_s that are much bigger than the electron Larmour radius, $\lambda_s \gg l_{eH}$. In such case the spin perturbation function is given by the first two moments:

$$\mu_B \delta S_j(\mathbf{p}, \mathbf{x}, t) = \frac{df^{(0)}}{d\varepsilon_p} \left[\int \frac{d^3p}{(2\pi)^3} \frac{df^{(0)}}{d\varepsilon_p}\right]^{-1} \times \left(m_j(\mathbf{x}, t) + \frac{3v_i}{v^2} \Sigma_{ij}(\mathbf{x}, t)\right) \ ,$$

that allows us to complete our hydrodynamical system by the equation for the pseudotensor Σ_{ij},

[2]The static susceptibility is small in a degenerate electron gas , $\chi_0 = \alpha v_{F_e}/4\pi^2 \ll 1$, in contrast with the varying one $\chi(t)$ (see below eq (20)). This is the reason why the static magnetic induction $B_0 = (1 + 4\pi\chi_0)H_0$ and the magnetic field strength H_0 practically coincide there.

$$\frac{\partial\Sigma_{ij}(\mathbf{x},t)}{\partial t}+\Omega_e\Big(e_{jtl}\hat{n}_l^B\Sigma_{it}(\mathbf{x},t)-e_{itl}\hat{n}_l^B\Sigma_{tj}(\mathbf{x},t)\Big)+$$
$$+\frac{<v^2>}{3}\frac{\partial}{\partial x_i}\Big(m_j(\mathbf{x},t)-\chi_0 b_j(\mathbf{x},t)\Big)-$$
$$-\frac{G_Fc_A\mu_Bn_{e0}}{\sqrt{2}m_e}\int\frac{d^3q}{(2\pi)^3}n_j(q)\frac{\partial\delta f^{(\nu)}(\mathbf{q},\mathbf{x},t)}{\partial x_j}=0\ . \tag{18}$$

Here $\Omega_e=eB_0/m_e$ is the electron cyclotron frequency;
$<v^2>=[\int d^3pdf^{(0)}/d\varepsilon_p]^{-1}\int d^3pv^2df^{(0)}/d\varepsilon_p$ is the average of the velocity squared; $\hat{n}^B=\mathbf{B}_0/B_0$ is the unit pseudovector and last true tensor term describes the parity violation through weak interactions. Note that eq. (18) occurs important accounting for the spatial dispersion and claims an inclusion of exchange interactions we omitted here.

IV. NEUTRINO DRIVEN STREAMING INSTABILITY OF SPIN WAVES

In this section we derive from generalized Bloch equation (17) the dispersion equation in magnetized plasma in the presence of neutrino beam.

Let us consider for simplicity the case of long-wave perturbations with the spectrum $\omega(\mathbf{k})$ obeying the inequalities

$$\frac{k^2}{\omega}\geq\omega\gg k<v>\ . \tag{19}$$

As the mean electron velocity $<v>$ is small in NR plasma, $<v>\ll 1$, and keeping in mind the Maxwell equation written in the Fourier representation $\mathbf{b}=[\mathbf{k}\times\delta\mathbf{E}]/\omega$ one finds from the condition $k/\omega\gg<v>$ that the electric field contribution in the spin RKE (3) ($\sim\delta\mathbf{E}$) is negligible comparing with the magnetic one even for the maximum frequency in whole space-like region $\omega\leq k$ relevant to the Čerenkov resonance with neutrinos, $\omega=\mathbf{kn}$. Without electric fields the transversal components of the permeability and the susceptibility tensors ($i,j=x,y$) appearing in the second term of (17) are diagonal in plasma, $\mu_{ij}(\omega,\mathbf{k})=\mu(\omega,\mathbf{k})\delta_{ij}$, $\chi_{ij}(\omega,\mathbf{k})=\chi(\omega,\mathbf{k})\delta_{ij}$, that differs this medium from ferromagnets.

Moreover, the latter inequality in (19), $\omega\gg k<v>$, means the high-frequency approximation, for which the spatial uniform susceptibility $\chi(t)$ and the permeability $\mu(t)=1+4\pi\chi(t)$ can be considered instead of more general ones, $\chi(\mathbf{x},t)$ and $\mu(\mathbf{x},t)$, that allows us to neglect the complicated pseudotensor term Σ_{ij} in (17) as well as the spatial dispersion in the perturbations $m_{\pm}(\mathbf{x},t)=\int dt'\chi(t-t')h_{\pm}(\mathbf{x},t')$ and $b_{\pm}(\mathbf{x},t)=\int dt'\mu(t-t')h_{\pm}(\mathbf{x},t')$ correspondingly where $h_{\pm}(\mathbf{x},t)=h_x(\mathbf{x},t)\pm ih_y(\mathbf{x},t)$ is the magnetic field strength perturbation.

Under such conditions we find from the linearized spin RKE (3) the susceptibility in QED plasma neglecting neutrinos,

$$\chi(\omega)=\frac{\pm\Omega_e\chi_0}{\omega\pm\Omega_e(1-4\pi\chi_0)}\ , \tag{20}$$

that may be finite at the paramagnetic resonance $\omega=\mp\Omega_e$ given by the equation $1+4\pi\chi(\omega)=0$. Latter follows from the shortened Maxwell equation $\mathbf{k}\times(\mathbf{b}-4\pi\mathbf{m})=0$ when one neglects electric field terms.

Then substituting into (17) the solution of the neutrino RKE (1) obtained in the same linear approximation (12),

$$\delta f^{(\nu)}(\mathbf{q},\mathbf{k},\omega)=\frac{G_Fc_V}{\sqrt{2}}\frac{k_k}{(\omega-\mathbf{kn})}\frac{\partial f_0^{(\nu)}(q)}{\partial q_k}\int\frac{d^3p_e}{(2\pi)^3}(1-\mathbf{v}_e\mathbf{n})\delta f^{(e)}(\mathbf{p}_e,\mathbf{k},\omega)+$$
$$+\frac{G_Fc_A}{2\sqrt{2}}\frac{k_kn_l(q)}{(\omega-\mathbf{kn})}\frac{\partial f_0^{(\nu)}(q)}{\partial q_k}\int\frac{d^3p_e}{(2\pi)^3}\delta S_l(\mathbf{p}_e,\mathbf{k},\omega)\ , \tag{21}$$

we can rewrite the generalized Bloch equation (17) in the Fourier representation as

$$-i\omega m_j(\mathbf{k},\omega)+2\mu_B[(\mathbf{m}(\mathbf{k},\omega)-\chi_0\mathbf{b}(\mathbf{k},\omega))\times\mathbf{B}_0]_j+\frac{c_A^2\Delta_\nu}{8}A_l^{(\nu)}(\omega,\mathbf{k})m_l(\mathbf{k},\omega)ik_j+$$
$$+\frac{c_Ac_V\Delta_\nu k_j}{32\pi m_e}\left(A_i^{(\nu)}(\omega,\mathbf{k})-\frac{B^{(\nu)}(\omega,\mathbf{k})}{\omega}k_i\right)\times$$
$$\times\Big(e_{ikl}k_k[b_l(\mathbf{k},\omega)-4\pi m_l(\mathbf{k},\omega)]+\omega\delta\mathbf{E}_i\Big)=0\ . \tag{22}$$

Here the factor Δ^{ν} is given after eq. (11); the vector $A_l^{(\nu)} \equiv A_l^{(\nu)}(\omega, \mathbf{k})$ and the scalar $B^{(\nu)} \equiv B^{(\nu)}(\omega, \mathbf{k})$ depend on the neutrino background distribution $\hat{f}_0^{(\nu)}(\mathbf{q})$,

$$A_l^{(\nu)} = \int \frac{d^3q}{(2\pi)^3}\frac{\hat{f}_0^{(\nu)}(\mathbf{q})}{q}\left(\frac{[(\mathbf{kn})^2 - k^2]n_l(q)}{(\omega - \mathbf{kn})^2} + \frac{(\mathbf{kn})n_l - k_l}{\omega - \mathbf{kn}}\right) , \tag{23}$$

$$B^{(\nu)} = \int \frac{d^3q}{(2\pi)^3}\frac{\hat{f}_0^{(\nu)}(\mathbf{q})}{q}\left(\frac{[(\mathbf{kn})^2 - k^2]}{(\omega - \mathbf{kn})^2}\right) . \tag{24}$$

Obtaining last term in eq. (22) ($\sim c_A c_V$) we used the exact Maxwell equation (6) when we substituted the *convection current* $\delta j_i(\mathbf{k}, \omega) = \int d^3p v_i \delta f^{(e)}(\mathbf{p}, \mathbf{k}, \omega)$ in the first line of eq. (21).

One can easily see that in NR plasma with a low electron density $n_{0e} \ll m_e^3 \sim 2\times 10^{31}~cm^{-3}$ the general condition of the macroscopic description $2\pi/k \gg (n_0)^{-1/3}$ means that long wave lengths exceeding the Compton one, $2\pi/k \gg m_e^{-1}$ are possible only, and due to (19) frequencies obey the inequality $\omega \leq k \ll m_e$.

Accounting for low ω and k and comparing the term $\sim c_V c_A$ with the previous one in eq. (22) ($\sim c_A^2$) one finds that the convection current gives a negligible contribution. Note that for $\nu_{\mu,\tau}$-neutrinos with $\mid c_V \mid \ll 1$ such correction becomes even less than for ν_e. Omitting also the small term $\sim \chi_0$ we arrive to the shortened form of (22),

$$-i\omega m_j(\mathbf{k}, \omega) + 2\mu_B[\mathbf{m}(\mathbf{k}, \omega) \times \mathbf{B}_0]_j + \frac{c_A^2 \Delta_\nu}{8} A_l^{(\nu)}(\omega, \mathbf{k}) m_l(\mathbf{k}, \omega) i k_j = 0 . \tag{25}$$

Thus, from the generalized Bloch equation (25) we have derived finally the dispersion equation in SM,

$$(\omega^2 - \Omega_e^2)\left(\omega - \frac{c_A^2 \Delta^{(\nu)}}{8} A_z^{(\nu)} k_z\right) - \frac{\omega^2 c_A^2 \Delta^{(\nu)}}{16}(A_-^{(\nu)} k_+ + A_+^{(\nu)} k_-) = 0 , \tag{26}$$

with the vector $A_i^{(\nu)}$ as given in eq. (23).

In the particular case of the neutrino beam, $\hat{f}_0^{(\nu)}(\mathbf{q}) = (2\pi)^3 \delta^{(3)}(\mathbf{q} - \mathbf{q}_0)$, $\mathbf{n}_0 = \mathbf{q}_0/q_0$, for the transversal neutrino propagation $\mathbf{n}_0 \perp \mathbf{B}_0$ with the beam direction along x-axis, $\mathbf{n}_0 = (1, 0, 0)$, and the magnetic field $\mathbf{B}_0 = (0, 0, B_0)$ we obtain from eq. (26) the resonant excitation of spin waves $\omega = \mathbf{kn}_0 + i\delta = \Omega_e + i\delta$ with the increment δ,

$$\delta \simeq \Omega_e^{1/3}\frac{\sqrt{3}}{4}\left(\frac{\Delta^{(\nu)}}{2q_0}\right)^{1/3}(\mid c_A \mid k \sin\theta_{q_0})^{2/3} \geq \Omega_e \frac{\sqrt{3}}{4}\left(\frac{\Delta^{(\nu)}}{2q_0}\right)^{1/3}(\mid c_A \mid \sin\theta_{q_0})^{2/3} , \tag{27}$$

where we substituted the scalar product $\mathbf{kn}_0 = k\cos\theta_{q_0} = k\sin\theta\cos\phi$ denoting θ_{q_0} as the angle between the neutrino beam direction and the wave vector $\mathbf{k}$; θ is the angle of $\mathbf{k}$ with respect to the magnetic field $\mathbf{B}_0$. Note that we relied in the last inequality on the frequency approximation $k \geq \omega \simeq \Omega_e$ assumed above.

Let us compare this increment with the fastest one in the case of isotropic plasma following from the first line in our eq. (11) [7],

$$\delta_{weak} = \frac{\sqrt{3}}{2}\omega_{pe}\left(\frac{\Delta_\nu}{q_0}\frac{\sin^6\theta_{q_0}}{\cos^4\theta_{q_0}}\right)^{1/3} . \tag{28}$$

One can easily see the advantage of the excitation of collective modes by the intense neutrino flux in a polarized electron gas. For a strong magnetic field in a dense plasma obeying $\Omega_e \gtrsim \omega_{pe}$ the angular dependence in eq. (27) $\sim (\sin\theta_{q_0})^{2/3}$ gives a less suppression of the increment for the small angles $\theta_{q_0} \leq \arccos(< v > /c)$ for which Landau damping for growing modes is absent. This is due to the absence of the factor $(1 - \omega^2/k^2)^2$ in the new dispersion equation (26). It is obvious that such angular dependence would be especially dangerous and important for the relativistic plasma case.

On the other hand, there are other arguments against streaming instability driven by neutrino beams in SN (e.g. not collimated beam) [10] to be relevant for neutrino propagation in a magnetized medium. Nevertheless, we have just showed that in a polarized electron gas the dispersion equations are quite different from the case of the isotropic plasma [7,10,18] that stimulates a future exploration of collective plasma phenomena in the presence of intense neutrino fluxes.

V. DISCUSSION AND CONCLUSIONS

The violation of parity in the SM lepton plasma given by axial vector currents ($\sim c_A$) leads to the growth of spin eigen modes through the excitation of them by the intense neutrino flux. These spin waves can be coupled with the magnetosonic ones analogously to the case of spin waves in ferromagnets [21] or could transfer their energy to electromagnetic and plasma waves at the cross of spectra that finally could lead to the heating of ions and the background plasma.

The possible explanation of the shock revival in SN by different collective mechanisms including the neutrino driven streaming instability seems to be very perspective goal for future studies in the case of polarized electron gas.

The application of these mechanisms in the magnetized plasma behind the shock and outside the SN neutrinosphere is self-consistent with plasma parameters expected there. Really, we do not consider neutrino collisions with matter within this region using the collisionless neutrino RKE (1) and may also neglect the electron-ion collision frequency ν_{ei} comparing with the cyclotron frequency at the paramagnetic resonance $\omega = \Omega_e$. Hence collisionless Vlasov approximation should be valid for the spin equation (3) as well as for the electron RKE (2) since the Debye number is large, $N_D = n_{0e} r_D^3 \gg 1$. Indeed, in the field $B_0 = 10^{12}\ Gauss$ the cyclotron frequency reaches $\Omega_e = 1.7 \times 10^7 B_0 \sim 1.7 \times 10^{19}\ sec^{-1}$. This frequency is comparable with the plasma one at the density $n_{e0} \sim 10^{29}\ cm^{-3}$, $\omega_{pe} = 1.8 \times 10^{19}\ sec^{-1}$, and occurs larger than e.g. the electron-proton collision frequency $\nu_{ep} = 50 n_{e0}(cm^{-3})/(T_e(K))^{3/2}\ sec^{-1} \sim 1.6 \times 10^{17}\ sec^{-1}$ in the surrounding NR plasma with the temperature $T_e \sim 10^9\ K$. For these parameters the Debye radius $r_D \sim 10^{-9}\ cm$ obeys the large plasma parameter $N_D \sim 100 \gg 1$.

Under such conditions for the mean neutrino energy $q_0 \sim 10\ MeV$ the increment (27) reaches the maximum value $\delta \sim 10^{10}\ sec^{-1}$ that means too sharp collimated neutrino beam with the spread of directions $\mathbf{n}_0$ for fixed k not exceeding $\Delta \mathbf{k}\mathbf{n}_0/\Omega_e \lesssim \delta/\Omega_e \sim 10^{-9}$. If neutrinos move along radii beyond the neutrinosphere $r > R_\nu$ this estimate is too optimistic for macroscopic scales $L > 10^{-9} r$ since there is a spread of the angular distribution of neutrino trajectories that damages simple model with parallel rays assumed here [10].

Nevertheless, we think the simple dispersion equation (26) based on the enhancement of pure magnetic field perturbations is only a particular case of the general kinetic equations derived here.

Note that even in this approximation there are some advantages of our model comparing with the isotropic plasma case discussed in [7,10,17,18] as we have just shown in previous section (after eq. (28)).

A more general case with the overlap of electromagnetic eigen modes in a polarized medium and accounting for the spatial dispersion in such plasma seems to be the more realistic model for a magnetized SN while it is beyond of the scope of the present work .

We would like to acknowledge discussions with A.S. Volokitin. This work was supported for V.S. by the RFBR grant 00-02-16271.

---

[1] V.N. Oraevsky, V.B. Semikoz, Physica **142A**, 135 (1987); V.N. Oraevsky, V.B. Semikoz, Ya.A. Smorodinsky, Sov. Phys. JETP Lett., **43**, 709 (1986).
[2] V.N. Oraevsky, V.B. Semikoz, Ya. A. Smorodinsky, Physics of Elementary Particles and Atomic Nucleus (review in ECHAYA, Dubna) **25**, 2, 312-376 (1994) (in Russian) [Sov. J. Part. Nucl. **25**, 1-5, 129-156 (1995)].
[3] J. F. Nieves and P. B. Pal, Phys. Rev. D **49**, 1398 (1994).
[4] V.B. Semikoz, Sov. J. Nucl. Phys. **46**, 946 (1987).
[5] L.B. Leinson, V.B. Semikoz, V.N. Oraevsky, Phys. Lett. B **209**, 157 (1988).
[6] V.B. Semikoz, Physica **142A**, 157 (1987).
[7] R. Bingham, J.M. Dawson, J.J. Su, H.A. Bethe, Phys. Lett. A **220**, 107 (1994); L.O. Silva et al., Phys. Rev. D **60**, 068701 (1999); L.O. Silva, R. Bingham, J.M. Dawson, J.T. Mendonca, P.K. Shukla, Phys. Rev. Lett. **83**, 2703 (1999).
[8] V. Semikoz, J.W.F. Valle, Nucl. Phys. **B425**, 651 (1994); erratum Nucl. Phys. **B485**, 545 (1997); S. Sahu, V. Semikoz and J.W.F. Valle, hep-ph/9512390; P. Elmfors, D. Grasso and G. Raffelt, Nucl. Phys. **B479**, 3 (1996); J.C.D'Olivo, J.Nieves, Phys. Lett. B **383**, 87 (1996);
[9] A. Kuzenko and G. Segre, Phys. Rev. Lett. **77**, 4872 (1996).
[10] L. Bento, Phys. Rev. D **61**, 013004 (2000).
[11] V.P. Silin in *Spin waves* by A.I. Akhiezer, V.G. Bary'achtar, S.V. Peletminsky (Amsterdam, North-Holland Pub. Co., 1968.).
[12] V.B. Berestetskiĭ, E.M. Lifshits, and L.P. Pitaevskiĭ, *Quantum Electrodynamics* (2nd ed., Pergamon Press, Oxford, 1982).
[13] V.B. Semikoz, Phys. Rev. D **48**, 5264 (1993).

[14] V.N. Oraevsky, A.Yu. Plakhov, V.B. Semikoz, Ya.A. Smorodinsky, Sov. Phys. JETP **66**, 890 (1987); V.N. Oraevsky, V.B. Semikoz, Ya.A. Smorodinsky, Erratum, Sov. Phys. JETP **68**, 1309 (1989); V.B. Semikoz, Ya.A. Smorodinsky, Sov. Phys. JETP **68**, 20 (1989); J.C. D'Olivo, J.F. Nieves, P.B. Pal, Phys. Rev. D **40**, 3679 (1989).
[15] E.M. Lifshitz and L.P. Pitaevskiĭ *Physical Kinetics* (Vol. 10 of Course of Theoretical Physics, Butterworth-Heinemann, Oxford, 1997), eqs. (28.1).
[16] V.N. Oraevsky, V.N. Ursov, Phys. Lett. B **209**, 83 (1988).
[17] S.J. Hardi and D.B. Melrose, Astrophys. J. **480**, 705 (1997); J.F. Nieves, Phys. Rev. D **61**, 113008 (2000); H.-Th. Elze, T. Kodama, R. Opher, Phys. Rev. D **63**, 013008 (2001).
[18] J.M. Laming, Phys. Lett. A **255**, 318 (1999).
[19] R. Kubo and T. Nagamiya, *Solid State Physics* (Mc'Grau Book Co. Inc., New York-Oxford-Toronto, 1969), p.491.
[20] F. Bloch, Phys. Rev. **70**, 460 (1946).
[21] A.G. Sitenko, V.N. Oraevsky, J. Phys. Chem. Solids **29**, 1783 (1968).